Development and validation of a free-piston engine generator numerical model

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Abstract

This paper focuses on the numerical modelling of a spark ignited free-piston engine generator and the model validation with test results. Detailed sub-models for both starting process and steady operation were derived. The compression and expansion processes were not regarded as ideal gas isentropic processes; both heat transfer and air leakage were taken into consideration. The simulation results show good agreement with the prototype test data for both the starting process and steady operation. During the starting process, the difference of the in-cylinder gas pressure can be controlled within 1 bar for every running cycle. For the steady operation process, the difference was less than 5% and the areas enclosed on the pressure–volume diagram were similar, indicating that the power produced by the engine and the engine efficiency could be predicted by this model. Based on this model, the starting process with different starting motor forces and the combustion process with various throttle openings were simulated. The engine performance during stable operation at 100% engine load was predicted, and the efficiency of the prototype was estimated to be 31.5% at power output of 4 kW.

1. Introduction

As an alternative to conventional engines, free-piston engine generator (FPEG) is a promising power generation system due to its simplicity and high thermal efficiency therefore has attracted considerable research interests recently [1–6]. It integrates a linear combustion engine and a linear electrical machine into a single unit. Combustion in the engine chambers drives the translator to reciprocate in an almost resonant way and the linear electric machine converts part of the mover's kinetic energy to electricity. The thermal efficiency was estimated to be up to 46% (including friction and compressor losses) at a power level of 23 kW and the research showed promising results with respect to engine performance and emissions [7].

However, as the piston motion of the FPE is not restricted by the crankshaft mechanism, the piston is free to move between its top dead centre (TDC) and bottom dead centre (BDC), therefore the piston is only influenced by the gas and load forces acting upon it. This induces to problems such as difficulties in engine start, misfire, unstable operation and overall complex control strategies [8]. As a result, there has not been any stably operating prototype reported by now. Despite the research specifically addressing the modelling of FPEG, most researchers tended to adapt simplified ideal models widely used in conventional engines to simulate FPEG. Moreover, since free-piston engines have specific operating characteristics compared to conventional engines, the validation of the free-piston engine model still needs to be acknowledged. Christopher M. Atkinson along with the co-researchers in West Virginia University developed an engine computational model with the combination of dynamic and thermodynamic analysis. The dynamic model consisted of an evaluation of the frictional forces and the load of the engine. The thermodynamic analysis consisted of an evaluation of each process that characterise the engine cycle based on the First Law of Thermodynamics. A time-based Wiebe function was used to calculate the heat release during the combustion process. The parameters used were based on test data collected from a running prototype, including in-cylinder pressure, displacement and velocity. A parametric investigation was also performed to predict the behaviour of the engine over a wide operating range [9–11].

Mikalsen and Roskilly presented a modelling investigation of a free-piston engine generator and they discussed the feasibility of the implemented models. The sub-models to simulate the in-cylinder combustion were based on existing single-zone models commonly used in conventional diesel engines. The output parameters of the model were validated against test data from a Volvo
TAD1240 six-cylinder, turbocharged diesel engine. The results showed that it was able to predict real trends of the free-piston engine for varying engine operating conditions [8].

Goldsborough et al. at Sandia National Laboratories analysed the steady-state operating characteristics of a free-piston engine using a zero-dimensional, thermodynamic model with detailed chemical kinetics, and heat transfer, scavenging, and friction sub-models. Hydrogen was used as fuel. The simulation identified the critical parameters affecting the engine performance, and suggested the limits of possible improvement compared to conventional internal combustion engines. However, validation of the free-piston engine model was difficult due to the limited experimental data available from their prototype [12].

Zuo et al. at Beijing Institute of Technology provided numerical simulation on piston motion. A time-based numerical model was developed in Matlab to define the piston motion profiles. Multi-dimensional gas flow in the scavenging process of the free-piston engine was studied based on the numerical simulation results. A wide range of design and operating options including stroke length, valve overlapping distance, operation frequency and charging pressure were investigated to evaluate their effects on the scavenging performance. The measured in-cylinder pressure and scavenging pressure were used as the boundary conditions for their model development [13,14].

Nemecek Pavel at Czech Technical University described modelling and control of a free-piston generator. The model was based on simplified thermodynamic processes. Assumptions of ideal gas and ideal reversible processes were adapted. Despite the simplifications of the model, simulations results showed good agreement with the real system. However, the development of more precise thermodynamic identification was suggested for further work [15].

Free-piston engine is commonly modelled using simplified zero-dimensional models. Most of the reported models used idealised close system processes, in which no heat or mass transfer was considered [16–18]. However, the actual system cannot be assumed as isentropic close system because when the engine usually operates at relatively low speed, the effects of heat transfer and gas leakage become significant and cannot be simply neglected. Meanwhile, when the charge temperature rises above the wall temperature, heat is transferring from the air to the wall, which affects the piston's dynamics as well [19]. Thus, the ideal gas relationship is not sufficiently accurate for the present modelling of the FPEG. Moreover, friction force in previous models was considered to be a constant value, which is not accurate either. Furthermore, there has not been any model validation reported due to the limited test data available from operating prototypes.

This paper focuses on a spark ignited free-piston engine generator. Detailed sub-models for both starting process and steady operation were derived and developed in Matlab/Simulink. The compression and expansion processes were not regarded as ideal gas isentropic process; both heat transfer and air leakage were taken into consideration. Model validation was undertaken with the test data from a running prototype, which showed good agreement. Using this model, the starting process and steady operation performance were analysed.

2. Model description

2.1. Holistic model structure

The numerical model primarily aims to precisely describe the piston motion which is governed by the Newton's second law. Therefore an engine dynamic model was developed on the top level. The specific forces acting upon the pistons are determined by the in-cylinder gas thermodynamic processes, mechanical friction force and linear electric machine force. Thus three sub-models

### Nomenclature

- \( A \): piston area (m²)
- \( A_{\text{cycl}} \): the in-cylinder surface area in contact with the gas (m²)
- \( A_{\text{leakage}} \): leakage area (m²)
- \( A_f \): friction parameter (–)
- \( D_f \): friction parameter (–)
- \( C \): capacitance of the circuit
- \( C_d \): discharge coefficient
- \( C_p \): heat capacity at constant pressure
- \( d \): cylinder diameter (mm)
- \( d_0 \): reference cylinder diameter
- \( E \): average temperature of lubrication oil at liner (°C)
- \( f \): overall scaling factor (–)
- \( F_e \): load force from the linear generator (N)
- \( F_f \): friction force (N)
- \( F_{g\ell} \): gas force from the left cylinder (N)
- \( F_m \): force output from the linear electric machine (N)
- \( F_r \): gas force from the right cylinder (N)
- \( F_{sr} \): gas force from the right scavenging pump (N)
- \( F_{tr} \): gas force from the left scavenging pump (N)
- \( H_{\text{eI}} \): enthalpy of the exhaust air (J)
- \( H_{\text{iI}} \): enthalpy of the intake air (J)
- \( H_{\text{lI}} \): enthalpy of the air leaked from the piston rings (J)
- \( h \): heat transfer coefficient (W/m² K)
- \( i \): current of the circuit (A)
- \( K_s \): proportionality constant for the thrust force of the motor
- \( K_r \): friction parameter (–)
- \( L \): inductance of the circuit
- \( m \): moving mass of the piston assembly
- \( m_{\text{air}} \): mass of the in-cylinder gas (kg)
- \( m_{\text{air}} \): mass flow rate of the in-cylinder gas (kg/s)
- \( P \): in-cylinder pressure (Pa)
- \( P_0 \): reference pressure or ambient pressure (Pa)
- \( P_r \): pressure in scavenge case (Pa)
- \( Q_c \): heat released from the combustion process (J)
- \( Q_{\text{in}} \): overall heat input for each cylinder in one running cycle
- \( Q_{\text{out}} \): heat transferred to the cylinder wall (J)
- \( Q_{\text{in}} \): transferred heat flow rate (J/s)
- \( R \): resistance of the circuit
- \( T_0 \): air temperature in the scavenging pump (K)
- \( T_w \): average temperature of the cylinder wall surfaces (K)
- \( t \): time at which the combustion process starts
- \( x \): mover’s displacement
- \( U \): internal energy of the in-cylinder gas (J)
- \( V \): instantaneous cylinder volume (m³)
- \( V_s \): volume of scavenge case (m³)
- \( \nu \): axial velocity of piston (m/s)
- \( \nu_p \): mean piston speed (m/s)
- \( \lambda \): fuel mass fraction burned
- \( \gamma \): ratio of heat capacities
- \( \phi \): magnetic flux
- \( \theta_0 \): reference temperature (°C)
that describe the abovementioned three groups of forces were developed on a lower level and the calculated forces are fed into the top level dynamic model to determine the piston motion.

The in-cylinder thermodynamic processes include compression or expansion process of the piston, heat transfer from the in-cylinder gas to the wall, gas leakage through the piston rings, and heat release of the combustion process. The scavenging process was also included since a two stroke engine was considered in this paper. The friction sub-model describes the friction force acting on the piston rings which is determined by a number of operating factors and is always a resistance force. The linear electric machine force however, can be either driving force or resisting force depending on its working mode. A schematic diagram of the model architecture is illustrated in Fig. 1.

The model was developed in Matlab/Simulink. It was calibrated by the parameters from a prototype. The input variables and output engine performance parameters are also demonstrated in Fig. 1. Each sub-model is enabled or disabled based on the piston displacement using State flow Chart function in Simulink. The equations were solved using Runge–Kutta solver with a fixed step size of $10^{-6}$.

This model was used for both starting process and steady operation investigation. In starting mode, the combustion sub-model was disabled and the linear electric machine was requested to work as a motor, i.e. providing driving force. In steady operation mode, the combustion sub-model was enabled and the linear electric machine works as a generator which provided a resistance force. It was assumed that at the end of starting process, the engine can be switched smoothly into steady operation mode. Therefore the calculated in-cylinder pressure, piston displacement and velocity at the end of the starting process were taken as the initial values for the steady operation simulation. The other constant input parameters for the simulation model are summarised in Table 1.

### 2.2. Engine dynamic model

This top level model considers the dynamics of the mover (piston assembly). The forces acting on the pistons include in-cylinder gas forces, linear motor force, mechanical friction force and the inertial force of the mover [8–14]. A dynamic equation of the mover can be derived from Newton’s Second Law and illustrated in Fig. 2.

$$\overline{F_l} + \overline{F_r} + \overline{F_o} + \overline{F_m} + \overline{F_f} = m \frac{d^2x}{dt^2}$$

(1)

where $x$ is the mover’s displacement; $m$ is the moving mass of the piston assembly; $F_l$ is the gas force from the left cylinder; $F_r$ is the gas force from the right cylinder, which is on the opposite direction of $F_l$; $F_o$ is the gas force from the right scavenging pump; $F_m$ is the gas force from the left scavenging pump. The gas forces are calculated from the gas pressure and piston area according to Eqs. (2) and (3); $F_m$ is the force output from the linear electric machine, and it varies from motoring (starting) and generation (steady operation) modes; $F_f$ is the friction force which is always on the opposite direction to the piston’s velocity. More detailed description of the above forces will be introduced in the following sections.

$$F_r = p_r \cdot A; \quad F_l = p_l \cdot A$$

(2)

![Fig. 1. Diagram of the simulation model.](image)

![Fig. 2. Dynamics of the piston.](image)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Simulation parameters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Value</td>
</tr>
<tr>
<td>Bore</td>
<td>52.5 mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>70.0 mm</td>
</tr>
<tr>
<td>Effective stroke length</td>
<td>35.0 mm</td>
</tr>
<tr>
<td>Moving mass</td>
<td>5.0 kg</td>
</tr>
<tr>
<td>Constant of back electromagnetic voltage</td>
<td>85.0 V/(m s$^{-1}$)</td>
</tr>
<tr>
<td>Thrust force constant</td>
<td>74.4 N/A</td>
</tr>
<tr>
<td>Coil resistance</td>
<td>14.0 $\Omega$</td>
</tr>
</tbody>
</table>
where $p$ is the pressure, $A$ is the piston area, and the subscripts $r$ and $l$ represent the values of the right and left cylinder.

2.3. In-cylinder thermodynamic sub-model

The in-cylinder gas was considered as ideal gas in closed system with corrections such as gas leakage and heat transfer. The analysis of the in-cylinder gas property during the compression and expansion phases of the cycle is based on a zero-dimensional, thermodynamic approach. When the intake or exhaust port is open during the gas exchange process, the in-cylinder pressure is assumed to be ambient pressure. Other important assumptions are: the in-cylinder gas exists as a homogeneous medium, uniform in temperature and composition; the in-cylinder gas kinetic and potential energy are neglected. The thermodynamic model is derived based on the energy conservation equations and the ideal gas equations.

2.3.1. Thermodynamic equations in compression and expansion processes

During compression and expansion processes, the models for both side cylinders are identical. A graphical demonstration of this thermodynamic system for one cylinder is shown in Fig. 3. The in-cylinder gas pressure is calculated by applying the First Law of Thermodynamics on the cylinder charge:

$$\frac{dU}{dt} = -p\frac{dV}{dt} + \left(\frac{dQ_c}{dt} - \frac{dQ_{ht}}{dt}\right) + \sum H_l - \sum H_r$$

(4)

where $U$ is the internal energy of the in-cylinder gas (J); $V$ is the volume of the cylinder ($m^3$); $Q_c$ is the heat released from the combustion process (J); $Q_{ht}$ is the heat transferred to the cylinder wall (J); $H_l$ is the enthalpy of the intake air (J); $H_r$ is the enthalpy of the exhaust air (J); $H_l$ is the enthalpy of the air leaked from the piston rings (J).

When the intake and exhaust ports are both closed, the energy conservation equation can be written as:

$$\frac{dU}{dt} = -p\frac{dV}{dt} + \left(\frac{dQ_c}{dt} - \frac{dQ_{ht}}{dt}\right) - \sum H_l$$

(5)

As the in-cylinder charge is assumed to be ideal gas, its internal energy is a function of temperature only, giving

$$U = m_{air}C_vT$$

(6)

The differential form of the equation above is derived as

$$dU = m_{air}C_vdT + C_vdm_{air}$$

(7)

where $m_{air}$ is the mass of the in-cylinder gas (kg); $C_v$ is the specific heat capacity at constant volume (J/kg K) that is considered constant through the temperature range; $T$ is the temperature of the in-cylinder gas (K).

As the in-cylinder gas follows the ideal gas equation,

$$pV = m_{air}RT$$

(8)

The ideal gas state equation is formulated in its differential form for further deriving the model,

$$pdV + Vdp = m_{air}RT + RTdm_{air}$$

(9)

Using the Mayer’s relation for the ideal gas,

$$C_p = C_v + R$$

(10)

where $C_p$ is the heat capacity at constant pressure, which is again considered constant through the temperature range.

The ratio of heat capacities is expressed as

$$\gamma = \frac{C_p}{C_v}$$

(11)

where $\gamma$ is the ratio of heat capacities.

The following correlation can be derived from Eqs. (4)–(11), which can be used to calculate the in-cylinder gas pressure during the compression, combustion and expansion processes.

$$\frac{dp}{dt} = \gamma - 1 \left(\frac{dQ_c}{dt} - \frac{dQ_{ht}}{dt}\right) - \frac{p}{m_{air}} \frac{dm_{air}}{dt} - \frac{pV}{V} \frac{dV}{dt}$$

(12)

2.3.2. Heat transfer

The in-cylinder charge temperature and the flow pattern vary enormously through the cycle. Both of these variables have a major influence on heat transfer. During the intake process, the intake charge is usually cooler than the walls and the flow velocity is high. During compression the charge temperature rises above the wall temperature, and gas velocity decreases, therefore heat is then transferred from the cylinder gas to the chamber walls [19]. The heat transfer between the cylinder walls and the in-cylinder gas is modelled according to Hohenberg [20]:

$$Q_{ht} = hA_{cy}l(T - T_w)$$

(13)

where $Q_{ht}$ is heat flow rate (J/s); $h$ is the coefficient of heat transfer ($W/m^2 K$); $A_{cy}$ is area of the in-cylinder surface in contact with the gas ($m^2$); $T_w$ is the average surfaces temperature of the cylinder wall (K).

The heat transfer coefficient is given by [20]

$$h = 130V^{-0.06} \left(\frac{P(t)}{10^4}\right)^{0.8} T^{-0.4}(\nu_p + 1.4)^{0.8}$$

(14)

where $V$ is the instantaneous cylinder volume ($m^3$); $\nu_p$ is the average piston speed ($m/s$).

2.3.3. Gas leakage

The mass flow rate through piston rings is described by the equation for compressible flow through a flow restriction. It is determined by temperature, composition, the in-cylinder gas pressure, gas pressure in the scavenging pump and a reference air leakage area. The mass flow rate equation is given as [19]:

$$\dot{m}_{air} = \frac{C_B \cdot \dot{A}_{leakage}}{(RT_0)^{1/2}} \left(\frac{P_l}{P(t)}\right)^{1/2} \left[\frac{2\gamma}{\gamma - 1}\left[1 - \left(\frac{P_l}{P(t)}\right)^{(\gamma - 1)/\gamma}\right]\right]^{1/2}$$

(15)

where $\dot{m}_{air}$ is the mass flow rate (kg/s); $C_B$ is the discharge coefficient; $\dot{A}_{leakage}$ is the leakage area ($m^2$); $T_0$ is the air temperature in the scavenging pump (K) and is assumed to be equal to the ambient temperature; $P_l$ represents the air pressure in the scavenging pump (Pa).

When the flow is choked,

$$P_l/P(t) \leq \left[2/(\gamma + 1)\right]^{(\gamma - 1)/\gamma}$$

(16)

The appropriate equation for mass flow rate becomes:
2.3.4. Combustion process

The simulation of the free-piston engine heat release in combustion process is one of the factors with the highest degree of uncertainty in this model. The piston motion of the FPEs differs significantly from that of conventional engines, and very little research result has been reported on how this influences the combustion process [8]. According to previous research, the energy released in the combustion is modelled using a modified Wiebe function [9]. Generally, the Wiebe function is related to the crankshaft angle, however this is not suitable for a linear engine. Therefore, a time based Wiebe function is used to express the mass fraction burned in the combustion process as:

\[
\dot{\lambda} = 1 - \exp\left(-a\frac{(t-t_s)}{C_d}^{b+1}\right) 
\]  

(18)

\[
dQc = \frac{Qin}{C_d} \frac{d\dot{\lambda}(t)}{dt} 
\]  

(19)

where \(\dot{\lambda}\) is the fuel mass fraction burned; \(a\) and \(b\) are shape factors, with the fitting value of 5 and 2 respectively [19]; \(C_d\) is the combustion duration with a constant value of 5 ms; \(t_s\) is the time at which the combustion process starts. \(Q_{in}\) is the overall heat input for each cylinder in one running cycle. Combining Eqs. (18) and (19), we have:

\[
\frac{dQc}{dt} = \frac{a + \frac{1}{C_d} + \frac{(t-t_s)}{C_d}}{b+1} \exp\left(-a\frac{(t-t_s)}{C_d}^{b+1}\right)Q_{in} 
\]  

(20)

Eq. (20) is used to predict the thermal energy delivered to the gas and the resulting pressure in the cylinder.

2.3.5. Scavenging process

Scavenging is considered part of the thermodynamic model for simplification. In fact, it is very difficult to represent the scavenging pressure using accurate numerical model, as it depends on the design details of the scavenging pump. However, estimating the scavenging performance is important to investigate its influence on engine performance, particular due to the variable stroke [14]. As a result, a simplified model is used to estimate the gas pressure in the scavenging pump. When both of the scavenging ports and intake port are covered, the gas mixture in the scavenging pump is assumed to go through an adiabatic process, and the equation for this is illustrated as

\[
\frac{dp_s}{dt} = -\gamma \frac{p_s}{V_s} \frac{dV_s}{dt} 
\]  

(21)

where \(p_s\) is the pressure in the scavenge case (Pa), \(V_s\) is the volume of the scavenge case (m³).

2.4. Linear electric machine model

Linear electric machines are electromagnetic devices capable of producing directly progressive unidirectional or oscillatory short-stroke motion. Linear electric generators are also linear motion electromagnetic devices which transform short-stroke oscillatory motion mechanical energy into single-phase AC electrical energy. Just as a rotary electric machine may operate either as a motor or as a generator, a linear electric machine may be switched to a generator to produce electricity in stable operation. As a result, two models are developed for the linear electric machine, namely the linear electric motor model and the linear electric generator model.

2.4.1. Linear electric motor for starting process

During the starting process, the linear electric machine operates as a linear motor. The static power to the linear motor is supplied by a three-phase AC source, which is specified in term of voltage and frequency: rated values and usual deviations. We use dq-model for the three phase linear synchronous motor. The dq-transformation for currents from the three phase abc variables is presented as [22]:

\[
\begin{bmatrix} I_d \\ I_q \\ I_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin \theta & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} 
\]  

(22)

The thrust force \(F_m\) can be obtained from the derivative of the magnetic stored energy, and is expressed as [23]

\[
F_m = K_A q_l
\]  

(23)

where \(K_A\) is the proportionality constant. It is determined by the characteristics of the permanent magnet of the mover. From Eq. (5), only \(q_l\) contributes to the thrust, which, when controlled, yields the desired mechanical characteristics [21].

In the linear motor control system, the motor is coupled with compensators to reduce the influence from back electromagnetic voltage. Using actual current as feedback signal, the injected current to the coil varies with the mover’s motion in order to maintain the output motor force as a constant value. The starting process is thoroughly investigated in another publication [24], and in the scale of this paper, the thrust force during starting process is a constant force determined by:

\[
F_m = \text{sign}(v) \cdot |F_m| 
\]  

(24)

2.4.2. Linear electric generator for steady operation

During steady operation, the linear electric machine operates as a generator. The translator of the generator forms part of the moving piston assembly. Through the continuous back and forth movement of the mover, the electrical current is generated from the alternator coils. According to linear generator fundamentals, the equivalent circuit of the generator and electrical load can be shown in Fig. 4.

The voltage across the generator \(e\) can be written as

\[
e = Ri + L \frac{di}{dt} + C \int idt 
\]  

(25)

where \(R\) is the resistance of the circuit; \(i\) is the current; \(L\) is the inductance of the circuit and \(C\) is the capacitance.

Assuming the load circuit is purely resistive (\(C = 0, L = 0\), Faraday’s electromagnetic induction laws give the back electromotive voltage \(e\) as:

\[
e = N \frac{d\phi}{dt} = k_e \frac{dx}{dt} 
\]  

(26)

where \(\phi\) is the magnetic flux and \(k_e\) is a generator property. It is assumed that the total mechanical energy of the piston movement
is converted to electricity, and there is no efficiency loss in the generator.

As the load force of the electric machine is assumed to be proportional to the current of the circuit, from Eqs. (25) and (26) the load force $F_e$ can be written as [12],

$$F_e = -c \nu$$

(27)

where $c$ is the load constant of the generator, which can be found from the physical parameters of the generator design specifications.

### 2.5. Friction sub-model

An analysis of engine friction mechanisms in four stroke spark ignition and diesel engines is presented by Heywood [19]. An approximate breakdown of rubbing and accessory friction is piston assembly 50%; valve train 25%; crankshaft bearings 10%; accessories 15% [19]. Friction work in the FPEG is expected to be lower than conventional internal combustion engines due to the elimination of the crank mechanism. Thus the friction in the wrist pin, big end, crankshaft, camshaft bearings, the valve mechanism, gears, or pulleys and belts which drive the camshaft and engine accessories is removed. Frictional losses of FPEG are mainly from the piston assembly, along with the linear electrical machine.

As there is no side forces act on the piston of FPEG and the movement of the piston is linear, piston assembly friction is dominated by the ring friction, and the friction from piston skirt is negligible. Thus the friction force of the FPEG is divided into three components, i.e. friction force from the linear electrical machine ($F_{fm}$) and friction forces between the piston rings and cylinder wall from both left ($F_{fr}$) and right ($F_{fr}$) cylinders of the engine. The total friction force is written as follows

$$F_f = F_{fm} + F_{fr} + F_{fr}$$

(28)

The friction of linear electrical machine comes from the contact of the mover and the stator. It is assumed to be constant as the velocity of the piston is low.

In the model (and prototype) each piston contains two compression rings and no oil ring. The initial tensions in both piston rings hold them out against the cylinder wall and hence generate friction. The in-cylinder gas pressure normally acts on the top and back of the rings and the pressure acting on the back of the rings increases this radial force and consequently the friction force. Correlations for piston ring friction have been developed by Bishop in the following categories: boundary condition friction (primarily between the rings and the cylinder wall due to ring tension, and gas pressure behind the compression rings) and viscous ring and piston friction. The component due to ring tension is essentially constant, and the component due to in-cylinder gas pressure behind the rings will vary depending on operation conditions.

Based on the discussion above, an empirical relationship is used to calculate the parameterized friction for the contact between the rings and the cylinder wall [25].

$$F_{fring} = f \left( -\text{sign}(\nu) \cdot A_j \cdot \sqrt{\nu} \cdot \left[ 1 - B_j \cdot \frac{E - \theta_0}{\theta_0} \right] \cdot \left[ 1 + K_v \cdot \frac{p(t)}{p_0} \right] \cdot \left( \frac{d}{d_0} \right) \cdot \frac{\phi_s}{\phi} \right)$$

(29)

where $f$ is the overall scaling factor ($\cdot$); $\nu$ is axial velocity of piston (m/s) and $\text{sign}(\nu)$ means the direction of piston velocity; $A_j$, $B_j$ and $K_v$ are all friction parameter ($\cdot$); $E$ is the average temperature of lubrication oil at liner (°C); $d$ is cylinder diameter (mm); $p$ is simultaneous in-cylinder pressure (bar); $\theta_0$ is reference temperature (°C); $p_0$ is reference pressure (1 bar); $d_0$ is reference cylinder diameter (165 mm).

### 3. Model validation

The model was validated for both starting process and stable operation against test data obtained from a prototype. The in-cylinder gas pressure, piston displacement and compression ratio were collected and calculated for analysis and the results of the comparison between simulation results and test data were demonstrated.

#### 3.1. Prototype test bench

The schematic FPEG prototype is shown in Fig. 5. This prototype configuration is identical with the input parameters used in the simulation. Further information of this prototype can be found in our previous publication [24]. When the engine is started by the linear electric machine by mechanical resonance method, the linear electric machine operates as a linear motor. It generates a constant motor force in the direction of the piston velocity, controlled by the aforementioned closed-loop strategy. The displacement amplitude as well as the peak in-cylinder gas pressure is expected to gradually grow and finally reach the required values for ignition. During the steady operation, the fuel delivery and ignition systems are activated and the electrical discharge between the spark plug electrodes starts the combustion process close to the end of the compression stroke. In this paper, the ignition timing is not optimised to the best performance.

#### 3.2. Validation results for the starting process

Fig. 6 shows the simulated in-cylinder gas pressure for the right cylinder during the starting process (noted as advanced model), compared with the test data at the same motoring force. As a comparison, an ideal model without considering gas leakage and heat transfer is demonstrated in the same figure (noted as ideal model). The fixed motor force acted on the mover is 110 N, which is in the direction of the mover’s velocity. In general, the maximum in-cylinder pressure grows and tends to reach a stable state after a few cycles. But the difference between the test results and ideal model simulation results is significant. For the ideal model, the compression and expansion processes are regarded as isentropic; the in-cylinder gas pressure is much higher and the maximum
value can reach 55 bars in 0.5 s. However, from the test data, the actual peak in-cylinder pressure grows to 12 bars during that period, which is almost the same value as the improved model predicted.

The advanced model shows a substantial improvement compared with the ideal model, and the prediction is much more accurate to the test results. The peak in-cylinder pressure for the first 6 cycles from the advanced model results and the test data are compared and plotted in Fig. 7 that shows a good agreement. Even though there is some difference in the frequency and the pressure value, the error can be controlled within a reasonable range (e.g., 1 bar for pressure). This can provide great confidence for the advanced model to be used for further investigation.

The compression ratios of the advanced model simulation and test results for both cylinders were calculated and presented in Fig. 8. For the first six cycles during the starting process, the compression ratio grows from 2:1 to nearly 8:1, which indicates the readiness for ignition. The simulation results for both left and right cylinders show similar trend with the test results, and the error of the advanced model can be controlled within 10%, indicating that the advanced model is valid and can predict the basic performance of the engine during the starting process.

3.3. Validation results for the steady operation

During the steady operation, the engine was operated at stoichiometric air–fuel ratio ($\lambda = 1.0$), and medium open throttle was applied. Fig. 9 shows the simulated pressure volume diagram for one running cycle, compared with the test results. From the results, the simulate model is of high accuracy to predict the actual engine performance. Both of the compression and expansion processes from the simulation are in good correspondence with the test results. The pressure difference is less than 5%, which is acceptable. Moreover, the areas enclosed by these two cycles are similar, which means that the work produced by the engine and the engine efficiency can be confidently predicted using this simulation model.

However it is necessary to point out, as the piston motion of FPEG is not restricted by a crankshaft-connection rod mechanism, the piston is free to move between its TDC and BDC, and the
movement is only controlled by the gas and load forces acting upon it. As a result, the peak in-cylinder gas pressure will differ from each cycle due to the cyclic variation introduced by many uncontrollable factors such as gas flow, mixture quality, spark quality and initial flame propagation. In order to validate the model, the in-cylinder gas pressure is an average of ten running cycles. Besides, in simulation, the in-cylinder gas pressure is regarded as ambient pressure once the exhaust port is opened, but it actually changes smoothly in the real prototype.

4. Simulation results and discussion

4.1. Starting force

The starting process was simulated with different motoring forces from 70 N to 130 N in 20 N interval. The peak in-cylinder gas pressure of the right cylinder for the first few running cycles is illustrated in Fig. 10. For a fixed motoring force with the proposed mechanical resonance starting strategy (see [24] for details of the mechanical resonance starting strategy), the peak in-cylinder gas pressure grows rapidly and reaches a stable value after about 10 cycles.

With an increased motoring force, the final stabilised peak pressure is higher and the peak pressure increase rate is greater. When 10 bar peak pressure is considered as the target, which is equivalent to a compression ratio of over 7:1, it takes 4 cycles to achieve with a motor force of 130 N, and 6 cycles with 110 N. However, with a motoring force of 90 N or less, the peak in-cylinder pressure cannot achieve the target. When the motoring force is 130 N, the peak pressure can even achieve 19 bar reasonably rapidly in less than half second, which is equivalent to a compression ratio of nearly 12:1 and considered sufficient for compression ignition engine start, since a low speed diesel engine usually has a compression ratio of approximately 14. This discussion can provide an indication for the linear electric machine selection to avoid oversizing but at the same time, ensure adequate motoring force can be generated to smoothly start the engine.

4.2. Engine load

The engine’s power output can be adjusted by varying the throttle opening. The effect of varied engine load on the engine performance was simulated to determine the optimal working conditions and corresponding control strategies. The engine was assumed in normal operation after warm up and a stoichiometric air/fuel mixture was supplied. The combustion process was assumed to be perfect and no loss was considered. Fig. 11 shows the simulated piston dynamics of the FPEG at varying engine load from 65% to 95% in 10% interval, and the engine performance with varied loads is summarised in Table 2.

The ignition timing was fixed at 28 mm from the central position regardless of the operation condition. It can be observed that the engine load has significant influence on the piston velocity and the engine stroke. Both the maximum piston velocity and the compression ratio vary in positive correlation with the load, with higher engine load corresponding to higher velocity and greater compression ratio.

Meanwhile, the velocity profile is relatively constant at the middle of the stroke, while changes greatly at BDC and TDC. The piston acceleration is much lower when piston approaching TDC/BDC than that when piston moving after TDC/BDC, mainly due to the combustion process. This also results in a high piston speed during the expansion process after TDC, which could be desirable in terms of heat transfer since the time is shorter when the gas temperature is high, therefore less heat loss is expected [12]. In addition, this particular feature of FPEG may lead to a reduction in the formation of temperature dependent emissions such as NOx.

4.3. Engine performance

The output engine performance parameters when the throttle is open at 100% are summarised in Table 3. The engine is running at 30 Hz (equivalent to 1800 rpm) which is low compared to the normal operation speed of a conventional spark-ignited engine. As a result, the maximum power output will be lower than a conventional engine at the same size. However, the estimated engine overall efficiency (including the friction loss, heat transfer loss, gas leakage loss, compression loss etc.) can reach 35% without turbocharger. As the efficiency of the linear electric generator is estimated to be more than 90%, the efficiency of FPEG can achieve 31.5% at a power level of approximately 4 kW. Please note the model was calibrated towards the bespoke prototype machine which has a relatively high gas leakage. For mass production engines with better quality control, the efficiency could be further improved.

Fig. 12 shows the simulated pressure–volume diagram under full load condition. It is suggested that the combustion is close to a constant volume heat release process, and the peak in-cylinder gas pressure can reach 80 bars. This can result in high efficiency. However unlike traditional engines, cyclic variations and imperfect
combustion in a free-piston engine may lead to significant variations in piston trajectory as the piston motion is not restricted by a crankshaft-connection rod mechanism. This induces to challenges to develop robust control strategies to ensure stable and smooth engine operation. As a result, the TDC must be controlled within a small range, as insufficient compression ratio may result in misfire, and overshoot may lead to mechanical contact between the piston and the cylinder head [26,27].

5. Conclusion

This paper introduced a complex model of a spark ignited free-piston engine generator. Detailed sub models for both starting process and steady operation process were derived. The simulation results showed a good agreement with the prototype test data for both the starting process and steady operation. During the starting process, the tested peak in-cylinder pressure grew to 12 bars in 0.5 s, which was almost the same as the numerical model predicted. The difference of the in-cylinder gas pressure history can be controlled within 1 bar. For the steady operation, the peak pressure difference was less than 5%, and the engine performances such as power and efficiency can be predicted at a high accuracy using this numerical model.

Using this model, the starting process at different motoring forces and the combustion process with varied throttle opening were investigated. The simulation results suggested that the peak in-cylinder gas pressure increased with higher motoring force, and a proper electric machine needs to be selected to match the engine to be able to provide sufficient motoring force and avoid oversizing at the same time. Both of the maximum piston velocity and the compression ratio varied in positive correlation with the throttle during steady operation. Higher throttle opening results in higher velocity and greater compression ratio. The velocity profile was relatively constant at the middle portion of the stroke, while changed greatly at BDC and TDC.

When the engine operated at 30 Hz (equivalent to 1800 rpm) during stable combustion process at full open throttle, the simulated combustion process was considered to be close to a constant volume heat release process, and the peak in-cylinder gas pressure reached 80 bars. The maximum power output of free-piston engine would be lower than a conventional engine of the same size. The estimated engine overall efficiency was 35% without a turbocharger at a power level of approximately 4 kW.

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References